

National Aeronautics and Space Administration



# CAUCHY DRAG ESTIMATION FOR LOW EARTH ORBITERS

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595 code

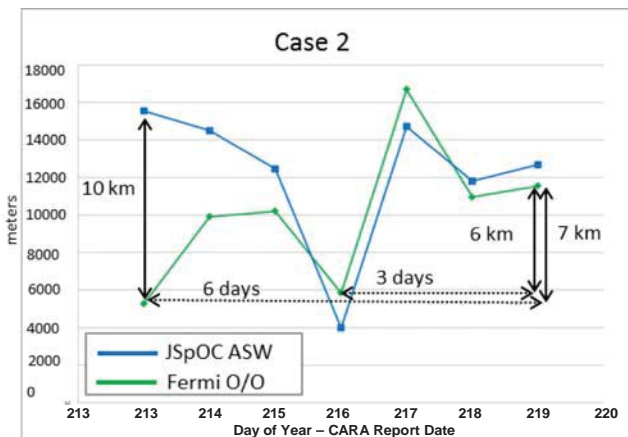
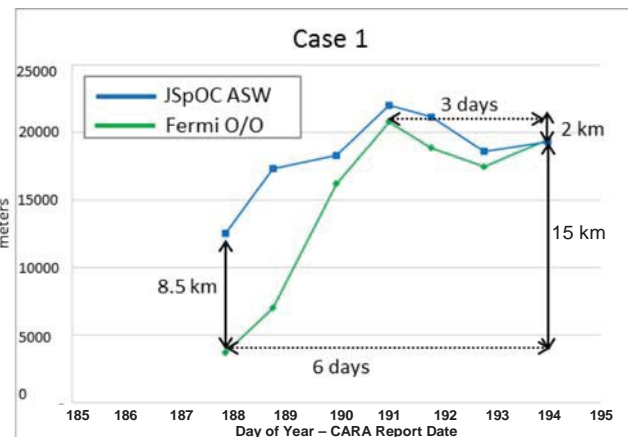


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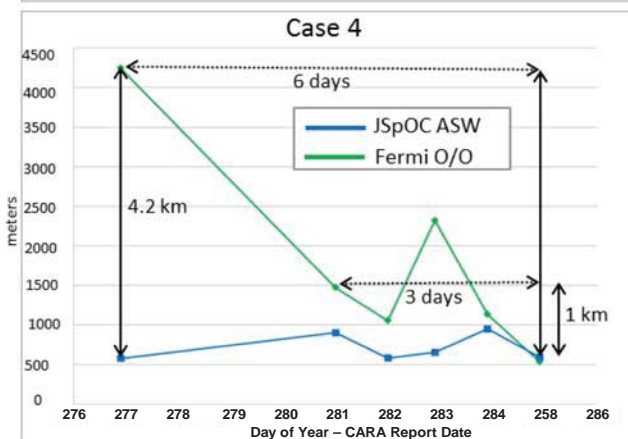
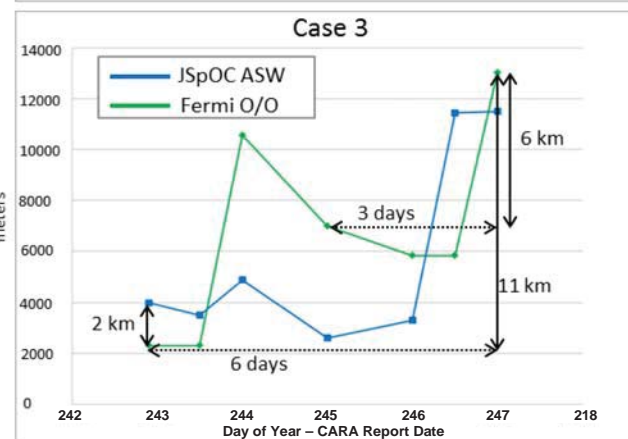
# GPS-based Owner/Operator Predictions are Sometimes Inferior to JSpOC's

Initial JSpOC prediction  
~2x closer to final consensus at Time of closest approach (TCA)



Initial JSpOC prediction  
~3x closer to final consensus at TCA

Initial predictions similar; GPS moving toward consensus sooner



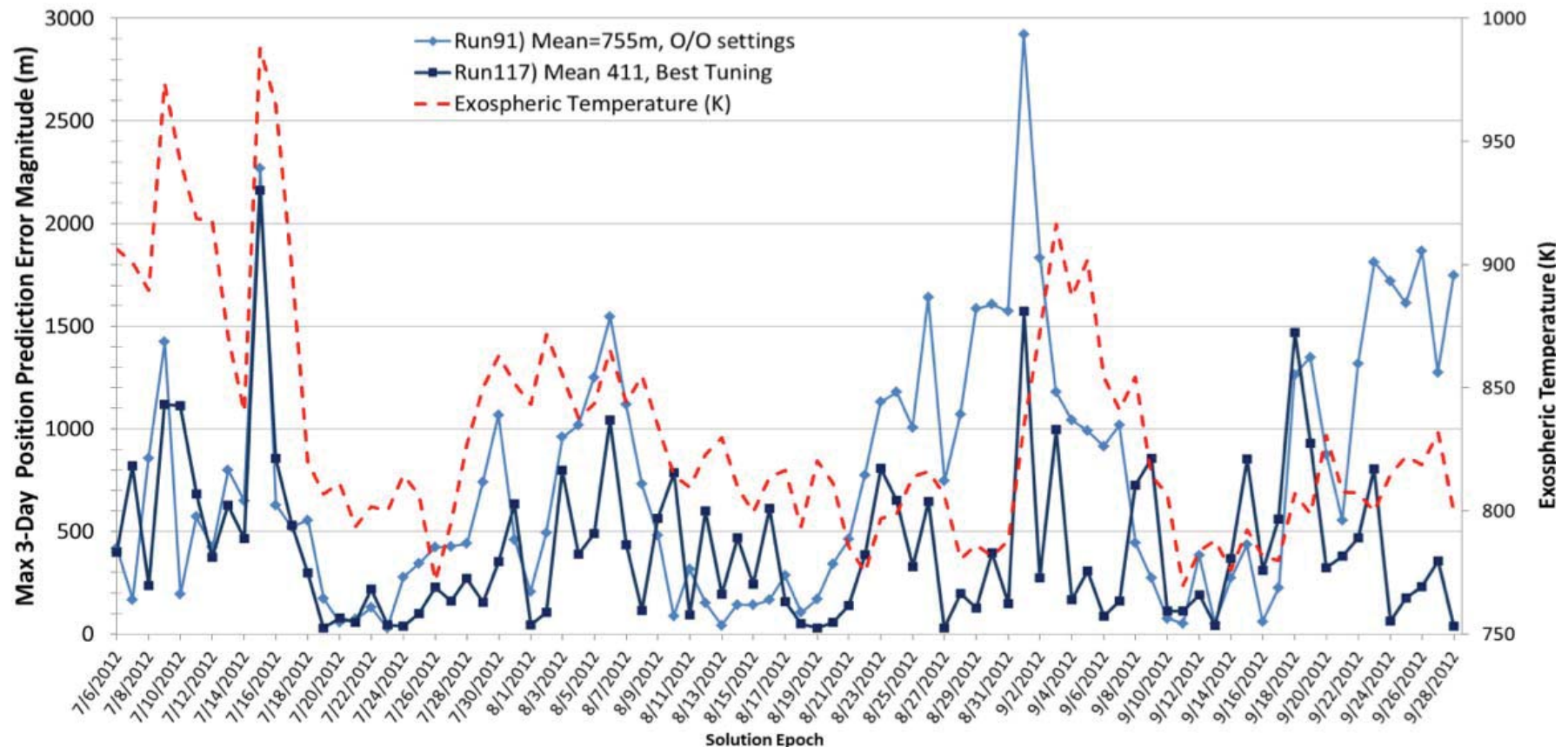
Initial JSpOC prediction within 500 of final consensus at TCA

Ref: M.A. Vavrina, C.P. Newman, S.E. Slojowski and J.R. Carpenter, "Improving Fermi Orbit Determination and Prediction in an Uncertain Atmospheric Drag Environment" Proceedings of the 24<sup>th</sup> International Symposium on Space Flight Dynamics, [www.issfd.org](http://www.issfd.org), 2014



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# Tuning GPS EKF Yields Marginal Improvement in Prediction Robustness to Density Variation



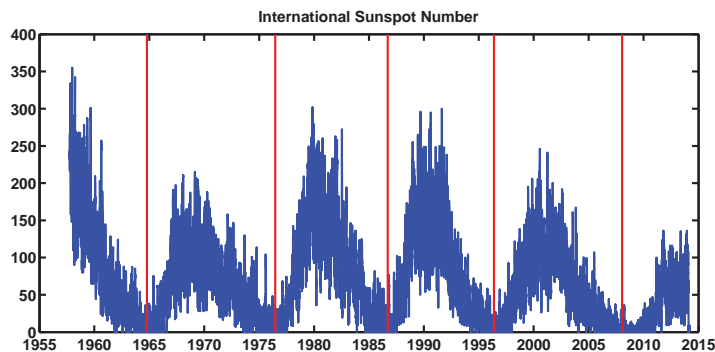
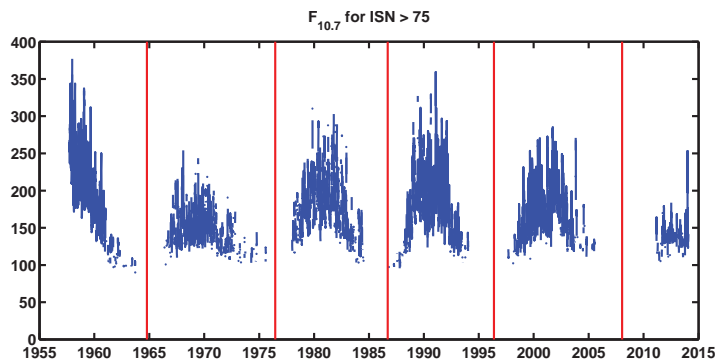
Ref: M.A. Vavrina, C.P. Newman, S.E. Slojowski and J.R. Carpenter, "Improving Fermi Orbit Determination and Prediction in an Uncertain Atmospheric Drag Environment" Proceedings of the 24<sup>th</sup> International Symposium on Space Flight Dynamics, [www.issfd.org](http://www.issfd.org), 2014



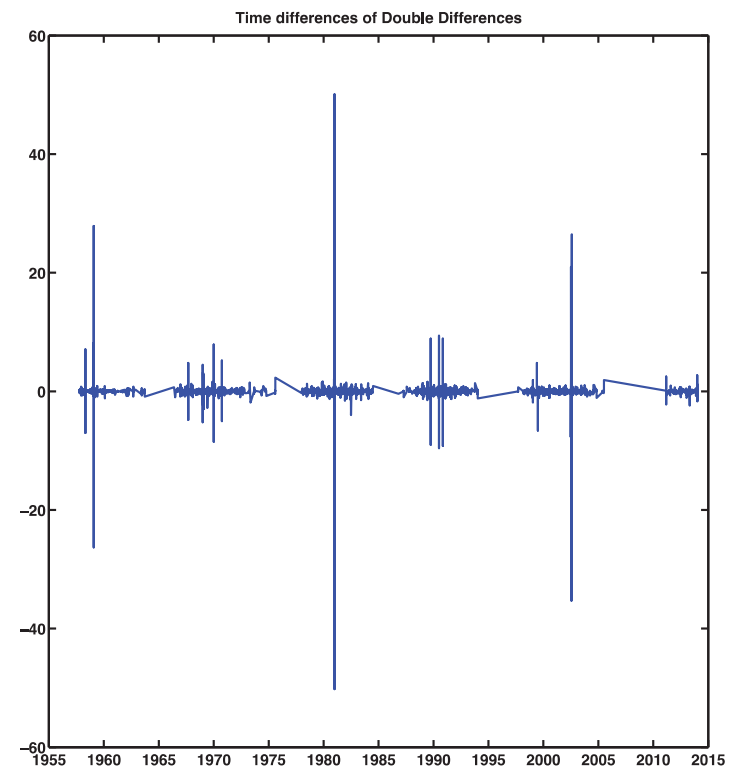
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# Solar Flux Variations During Higher Solar Activity Intervals are Not Gaussian

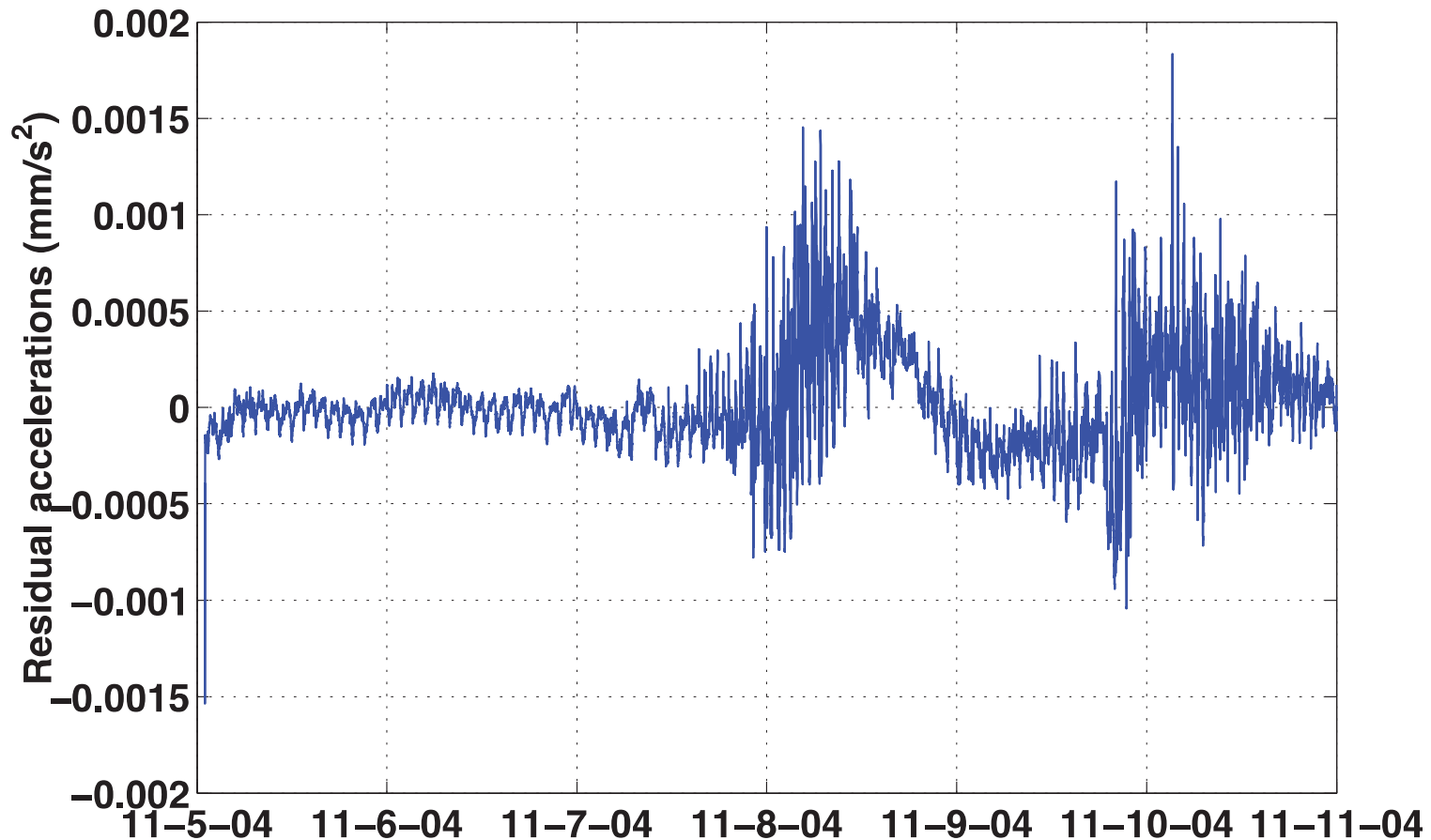
Data Set: Observed  $F_{10.7}$  Flux when ISN > 75



Variations in Data Set: Time differences of  
(Daily - Oct 84 day) - (Daily - Oct 84 day)

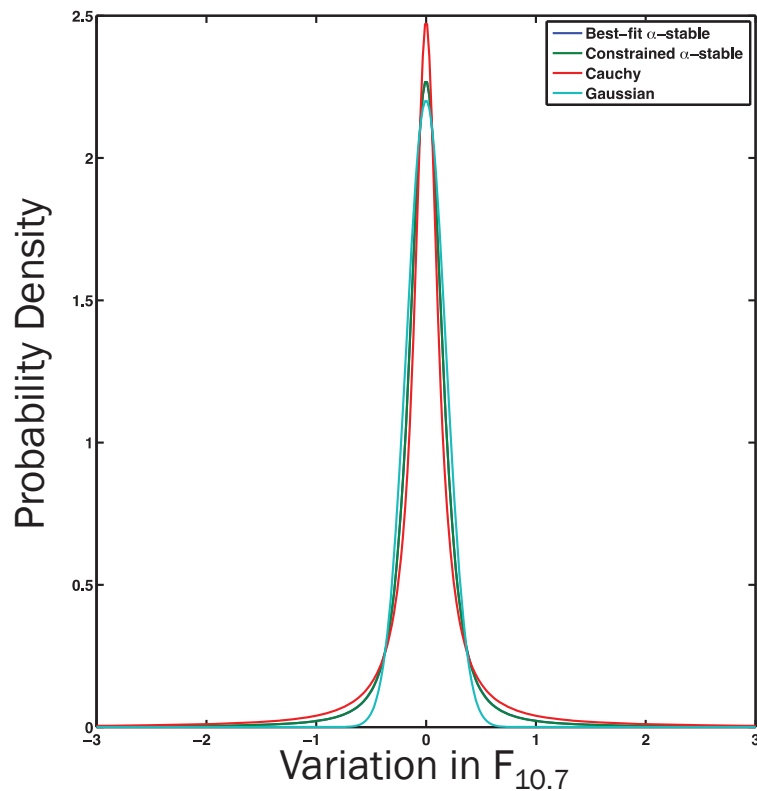


# Drag Residuals from CHAMP are Not Gaussian



# Fits to Stable Distributions Suggest a Cauchy Model

Fit PDFs for  $F_{10.7}$  Data



Stable Distribution Fits

Data	Concentration*	Asymmetry	Scale
$F_{10.7}$	1.38	0.02	0.13
$A_p$	1.03	0.0014	3.46
CHAMP drag	1.34	0.6121	0.0001

\*Gaussian = 2.0, Cauchy = 1.0



# The Idan-Speyer Scalar Cauchy Estimator (ISCE)

- Given a linear scalar system

$$x_{k+1} = \phi_k x_k + w_k$$

$$y_k = H_k x_k + v_k$$

- With Cauchy inputs and initial condition

$$p_{x_0}(x_0) = \frac{\alpha/\pi}{(x_0 - \bar{x}_0)^2 + \alpha^2}$$

$$p_{w_k}(w_k) = \frac{\beta/\pi}{w_k^2 + \beta^2}$$

$$p_{v_k}(v_k) = \frac{\gamma/\pi}{v_k^2 + \gamma^2}$$

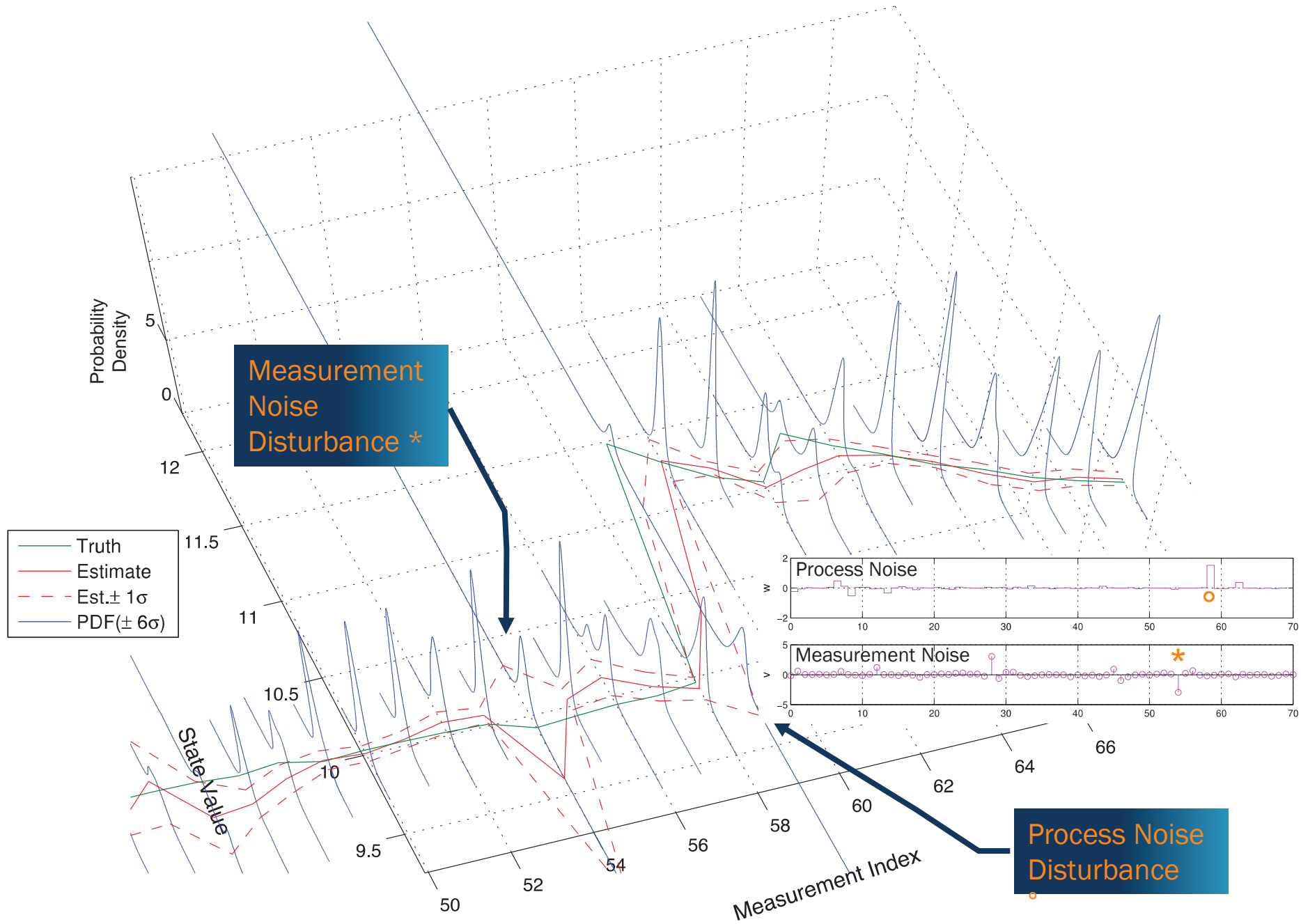
- Posterior PDF

$$p_{x_k|\mathbb{Y}_k}(x_k|\mathbb{Y}_k) = \sum_{i=1}^{k+2} \frac{a(i)_{k|k} x_k + b(i)_{k|k}}{(x_k - \sigma(i)_{k|k})^2 + \omega(i)_{k|k}^2}$$

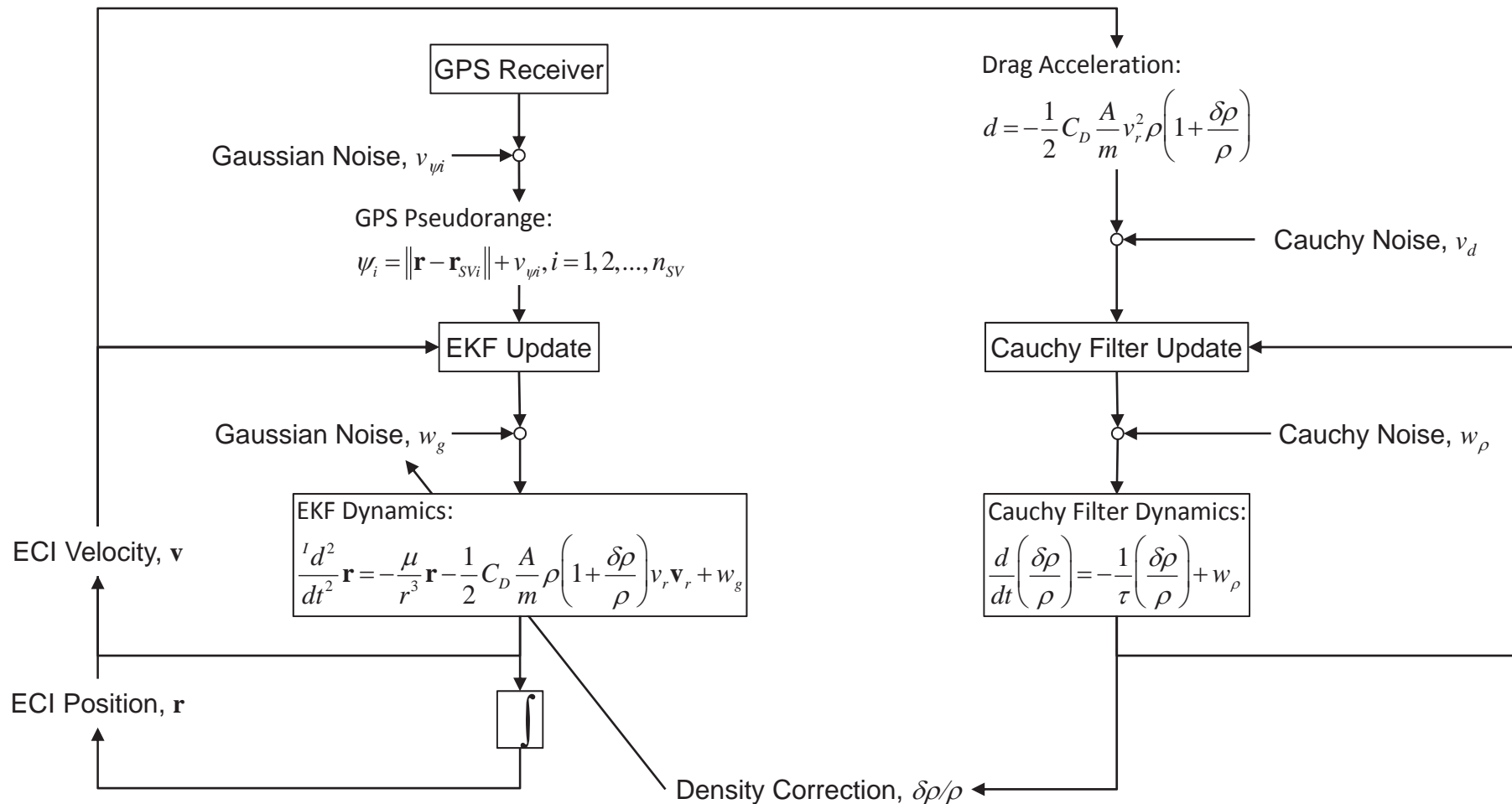
- Has Finite Moments

$$\begin{aligned} \hat{x}_{k|k} &= E[x_k|\mathbb{Y}_k] = \int_{-\infty}^{\infty} \xi_k p_{x_k|\mathbb{Y}_k}(\xi_k|\mathbb{Y}_k) d\xi \\ &= \pi \sum_{i=1}^{k+2} \frac{a(i)_{k|k} \left( \sigma(i)_{k|k}^2 - \omega(i)_{k|k}^2 \right) + b(i)_{k|k} \sigma(i)_{k|k}}{\omega(i)_{k|k}} \end{aligned}$$





# The ISCE May Be Embedded Within the EKF for Density Estimation



# Schmidt-Kalman Consider States Encapsulate the ISCE Moments

$$K_s = \begin{bmatrix} P_{ss_{k|k-1}} & P_{sc_{k|k-1}} \end{bmatrix} \begin{bmatrix} H_s^\top \\ H_c^\top \end{bmatrix} \left( \begin{bmatrix} H_s & H_c \end{bmatrix} \begin{bmatrix} P_{ss_{k|k-1}} & P_{sc_{k|k-1}} \\ P_{cs_{k|k-1}} & P_{cc_{k|k-1}} \end{bmatrix} \begin{bmatrix} H_s^\top \\ H_c^\top \end{bmatrix} + R_k \right)^{-1}$$

Solve-For States:  $\hat{s}_{k|k} = \hat{s}_{k|k-1} + K_s \left\{ z_k - h \left( \begin{bmatrix} \hat{s}_{k|k-1} \\ \hat{c}_{k|k-1} \end{bmatrix} \right) \right\}$

Consider States:  $\hat{c}_{k|k} = \hat{x}_{k|k} = \text{ISCE mean}$

$$P_{ss_{k|k}} = (I - K_s \begin{bmatrix} H_s & H_c \end{bmatrix}) P_{ss_{k|k-1}} - K_s H_c P_{cs_{k|k-1}}$$

$$P_{sc_{k|k}} = P_{cs_{k|k-1}}^\top = (I - K_s \begin{bmatrix} H_s & H_c \end{bmatrix}) P_{sc_{k|k-1}} - K_s H_c P_{cc_{k|k-1}}$$

$$P_{cc_{k|k}} = p_{x_{k|k}} = \text{ISCE variance}$$

$$\begin{bmatrix} \hat{s}_{k+1|k} \\ \hat{c}_{k+1|k} \end{bmatrix} = \int_{t_k}^{t_{k+1}} f \left( \begin{bmatrix} \hat{s}(\tau) \\ \hat{c}(\tau) \end{bmatrix} \right) d\tau$$

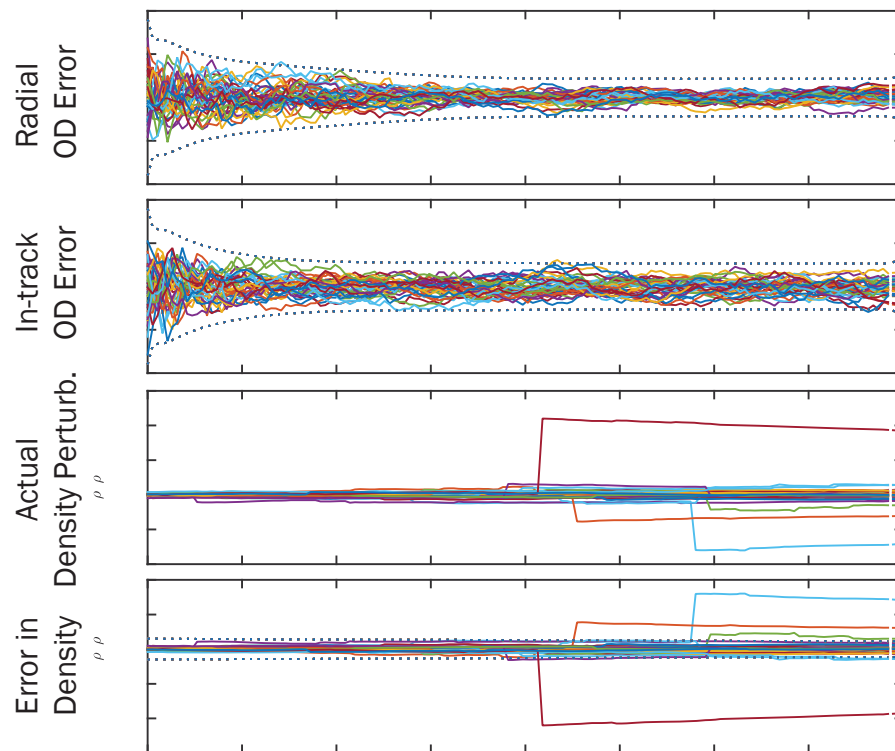
$$\begin{bmatrix} P_{ss_{k+1|k}} & P_{sc_{k+1|k}} \\ P_{cs_{k+1|k}} & P_{cc_{k+1|k}} \end{bmatrix} = \Phi(t_{k+1}, t_k) \begin{bmatrix} P_{ss_{k|k}} & P_{sc_{k|k}} \\ P_{cs_{k|k}} & P_{cc_{k|k}} \end{bmatrix} \Phi(t_{k+1}, t_k)^\top + Q(t_{k+1}, t_k)$$



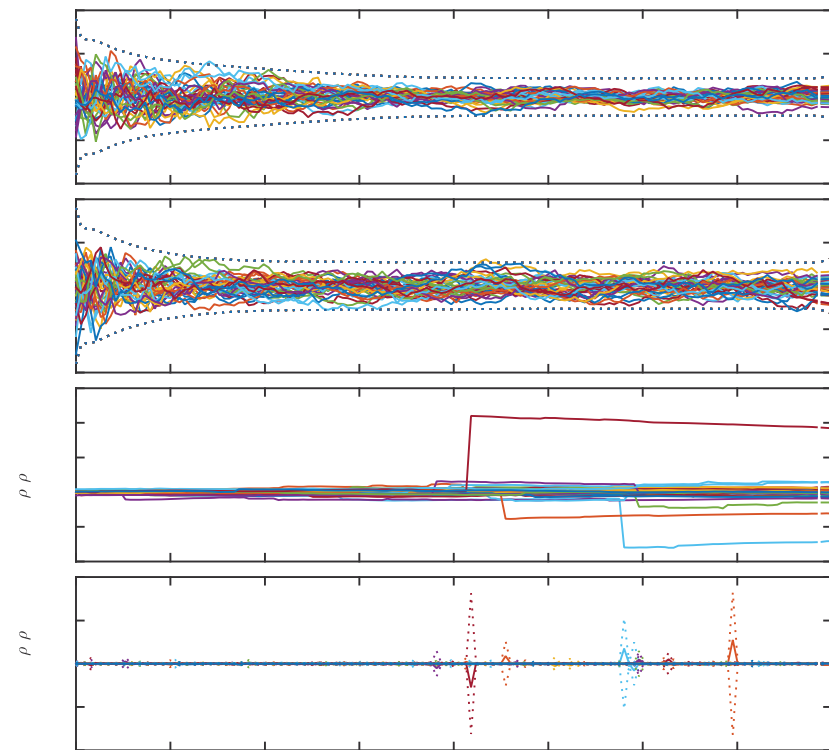
# Definitive OD is Similar, and Density Estimation is Superior (36 trials)

Dashed lines =  $\pm 3\sigma$  formal error

## Baseline EKF



## EKF Disciplined by ISCE



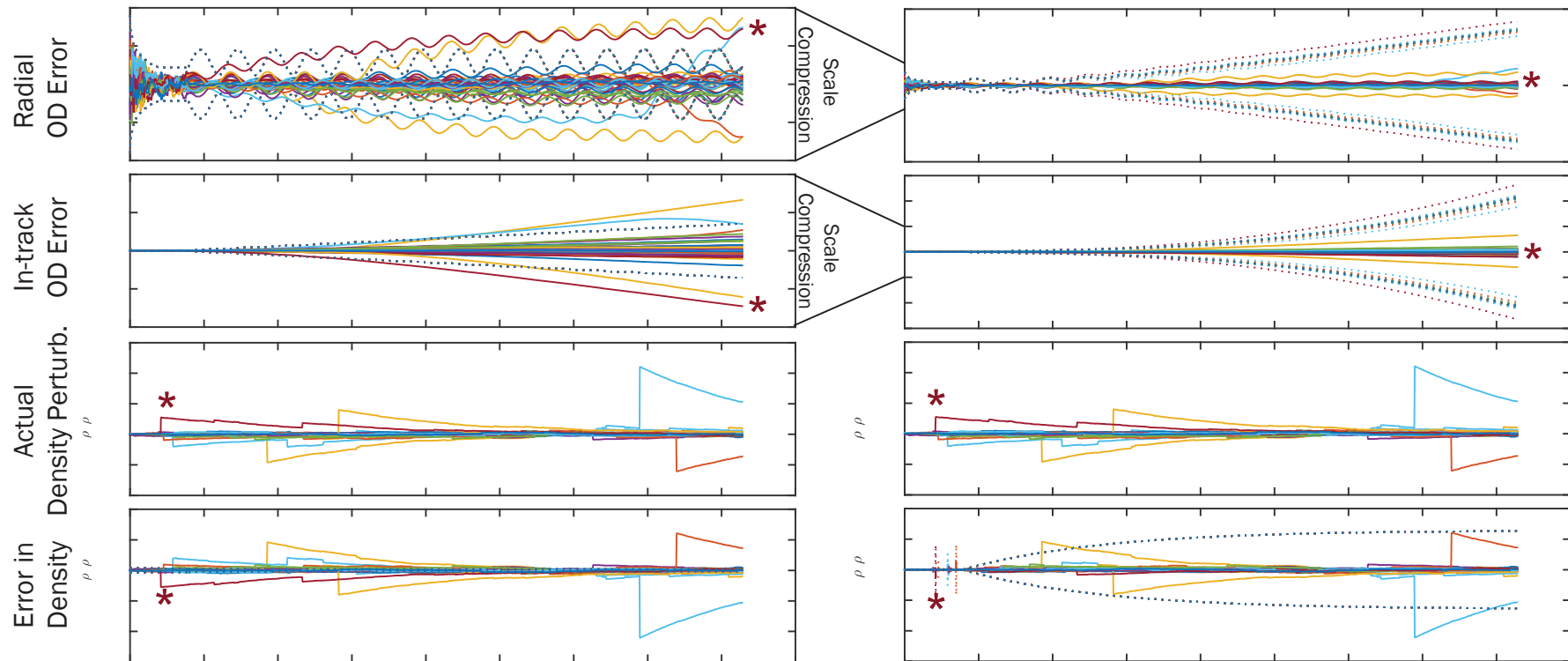
# Predictive OD: Superior for Observed, and Robust to Unobserved, Density Dispersions

Dashed lines =  $\pm 3\sigma$  formal error

\* Observed during definitive span

## Baseline EKF

## EKF Disciplined by ISCE



# Conclusions

- Space weather data show heavy-tailed characteristics that are better modeled by Cauchy than Gaussian
- Cauchy estimator (ISCE) may be embedded in EKF, using Schmidt-Kalman consider framework, for density estimation
- Definitive OD performance indistinguishable from EKF
- Predictive OD performance superior to EKF

